

LETTERS AND COMMENTS

Note on the meaning and terminology of Special Relativity

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Abstract. Special Relativity is a classical and simple theory. In spite of this, the pedagogical literature on it is still not free from confusion. This confusion is caused by the frequent usage of archaic notions, notations and equations, which have nothing to do with the essence of the theory, only with the history of its development. The aim of this short comment is to explain the basic notions of Special Relativity and to help the beginner to recognize and avoid the confusing notations and equations.

1. The relativity of Galileo Galilei

According to the principle of relativity, the relative motion of inertial reference frames cannot be detected by any experiments within these frames. The essence of relativity is expressed by the ‘gedanken ship’ described by Galileo Galilei in his famous book the *Dialogo*, published in Florence in 1632 [1]:

‘Shut yourself up with some friend in the main cabin below decks on some large ship, and have with you there some flies, butterflies, and other small flying animals. Have a large bowl of water with some fish in it; hang up a bottle that empties drop by drop into a wide vessel beneath it. With the ship standing still, observe carefully how the little animals fly with equal speed to all sides of the cabin. The fish swim indifferently in all directions; the drops fall into the vessel beneath; and, in throwing something to your friend, you need throw it no more strongly in one direction than another, the distances being equal: jumping with your feet together, you pass equal spaces in every direction. When you have observed all these things carefully (though there is no doubt that when the ship is

standing still everything must happen in this way), have the ship proceed with any speed you like, so long as the motion is uniform and not fluctuating this way and that, you will discover not the least change in all the effects named, nor could you tell from any of them whether the ship was moving or standing still. In jumping, you will pass on the floor the same spaces as before, nor will you make larger jumps toward the stern than toward the prow even though the ship is moving quite rapidly, despite the fact that during the time you are in the air the floor under you will be going in a direction opposite to your jump. In throwing something to your companion, you will need no more force to get it to him whether he is in the direction of the bow or the stern, with yourself situated opposite. The droplets will fall as before into the vessel beneath without dropping toward the stern, although while the drops are in the air the ship runs many spans. The fish in their water will swim toward the front of their bowl with no more effort than toward the back, and will go with equal ease to bait placed anywhere around the edges of the bowl. Finally the butterflies and flies will continue their flights indifferently toward every side, nor will it ever happen that they are concentrated toward the stern, as if tired

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out from keeping up with the course of the ship, from which they will have been separated during long intervals by keeping themselves in the air. And if smoke is made by burning some incense, it will be seen going up in the form of a little cloud, remaining still and moving no more toward one side than the other. The cause of all these correspondences of effects is the fact that the ship's motion is common to all the things contained in it, and to the air also. That is why I said you should be below decks; for if this took place above in the open air, which would not follow the course of the ship, more or less noticeable differences would be seen in some of the effects noted.'

All laws of nature and hence equations of motion and observable effects look the same in reference frames moving rectilinearly with constant velocity relative to the stars. In order to preserve the form of the Newtonian equations the time and space coordinates of two inertial frames should obey the relations

$$t' = t \quad (1a)$$

$$x' = x + vt \quad (1b)$$

$$y' = y \quad z' = z \quad (1c)$$

where v is the velocity of the ship, which moves along the x -axis, primed coordinates refer to the frame of the shore, and unprimed to that of the ship.

2. The relativity of Albert Einstein

The discovery of Maxwell's equations and the necessity of combining them with the equations of mechanics has led to modification of Newton's equations and to the Lorentz transformations:

$$t' = \gamma \left(t + \frac{vx}{c^2} \right) \quad (2a)$$

$$x' = \gamma(x + vt) \quad (2b)$$

$$y' = y \quad z' = z \quad (2c)$$

where

$$\gamma = \left(1 - \frac{v^2}{c^2} \right)^{-1/2} \quad (3)$$

and c is the maximum velocity a particle (body) may have in a vacuum, i.e. the velocity

of light. Michelson's experiment showed that relative motion of inertial frames cannot be detected even if one uses an optical interferometer. The interference pattern of two perpendicular rays of light did not depend on the orientation of the interferometer.

One may say that Einstein brought Michelson's interferometer onto Galileo's ship. In the course of the 20th century it was proved that not only mechanical, optical and electromagnetic phenomena, but also all atomic and nuclear interactions are invariant under the Lorentz transformations (2). At $v/c \ll 1$, $\gamma \rightarrow 1$ and equations (2) coincide with equations (1). According to equations (2) not only the velocities of bodies on the ship, but also the velocity of the ship itself may be of the order of the velocity of light.

The new feature of equation (2) was that time was no longer universal: it became relative, i.e. dependent on the relative velocity of the reference frames. The word relativity acquired several new meanings. In particular particles with velocities close to c are called relativistic. According to relativity theory the momentum $\mathbf{p}(p_x, p_y, p_z)$ of a particle and E/c^2 , where E is its energy, transform similarly to $\mathbf{r}(x, y, z)$ and t :

$$E' = \gamma(E + vp_x) \quad (4a)$$

$$p'_x = \gamma \left(p_x + \frac{vE}{c^2} \right) \quad (4b)$$

$$p'_y = p_y \quad p'_z = p_z. \quad (4c)$$

It is easy to see from equations (2) that the quantity

$$\tau^2 = t^2 - \frac{r^2}{c^2} \quad (5)$$

is invariant under Lorentz transformation:

$$\tau'^2 = \tau^2. \quad (6)$$

The physical meaning of τ is that of time in the rest frame of the particle, the proper time of the particle. Another important invariant is the mass m of the particle, its 'proper mass':

$$m^2 = \left(\frac{E}{c^2} \right)^2 - \frac{\mathbf{p}^2}{c^2}. \quad (7)$$

A great discovery, the one that Einstein made in 1905 [2], was that a massive body at rest still had energy, which he denoted by

E_0 and called rest energy. As follows from equation (7)

$$E_0 = mc^2. \quad (8)$$

The rest energy plays a unique role in all types of energy transformations: from nuclear to chemical reactions, as explained in many books and articles. If we apply equations (4) to a particle in its rest frame, in the laboratory frame (where the particle is seen to move with velocity v) we obtain

$$E = mc^2\gamma = \gamma E_0 \quad (9a)$$

$$p = m\gamma v. \quad (9b)$$

Here we may forget about the ship and consider v as the velocity of the particle. Equation (9a) may be considered as another definition of γ :

$$\gamma = \frac{E}{mc^2}. \quad (10)$$

Note that for $v \ll c$ we obtain from (9)

$$T = E - mc^2 \simeq \frac{1}{2}mv^2 \quad (11a)$$

$$p \simeq mv \quad (11b)$$

where T is the non-relativistic kinetic energy, while p is the non-relativistic momentum. Equations (11) stress that in all equations m is the usual Newtonian mass. A consistent presentation of Special Relativity can be found in [3–5].

3. Archaic notions and notations

Many years ago, when Special Relativity was in the process of development and experimental verification, many notions and kinds of notation were introduced which were later abandoned in favour of those which are described in the first two sections of this comment. Unfortunately some of these outdated notions are still being promoted by authors of many textbooks, popular books and articles.

One of the main sources of confusion is the use of the non-relativistic relation between momentum and velocity in place of the relativistic one (equation (11b) instead of equation (9b)).

As a result m in (11b) plays the role of $m\gamma$ in (9b). This leads to the false conclusion that

the mass of a body increases with its velocity. This ‘mass’ (which is actually E/c^2) is called relativistic mass. This leads immediately to the ‘most famous formula’

$$E = mc^2 \quad (12)$$

which according to many authors is the central formula of Special Relativity. Many famous physicists, when writing popular texts, used this formula (probably because it is so famous and hence self-promoting) instead of the correct formula (8c): $E_0 = mc^2$. (Einstein himself alternated between (12) and (8b), although he preferred (8b). Sometimes he used the relation $E = mc^2$ in the following sense: if E is the energy of radiation emitted or absorbed by a massive body at rest, then m is the change (negative, or positive, respectively) of its mass.)

If one uses m to denote E/c^2 one has to introduce the so called ‘rest mass’ m_0 to denote the Newtonian mass:

$$m = m_0\gamma. \quad (13)$$

For experts in relativistic physics the misuse of (11b) and use of equations (12) and (13) do not present any difficulty in understanding texts which contain these pseudo-relativistic equations. However, in the minds of beginners and outsiders they create confusion, and misunderstanding, and produce wrong intuition.

A decade ago an effort was made [5, 6] to explain the origin and the danger of archaic terminology. As a result recent editions of many university and college textbooks have dropped it, and use only rational modern terminology. However, some are still promoting confusion [7]. Some papers published recently by *European Journal of Physics* are also based on confusing terminology. The Editorial Board of the journal decided to publish this Comment in order to rectify this deplorable situation.

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